Applied Problems with Discipline Specific Formulae

BIOLOGY

Problem 1.A biological scientist working in a lab frequently uses the formula

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

They use instruments to measure density and mass of a solution but need to find the volume. Rearrange the formula to help the biologist compute the volume.

Problem 2. A *reagent* is a substance or compound added to a solution to cause a chemical reaction, or added to test if a reaction occurs. When working with reagents a biological scientist calculates the concentration of a diluted reagent using the formula:

$$C_1V_1 = C_2V_2$$

where C_1 is the initial concentration of the stock solution, V_1 is the amount of stock solution taken to perform the dilution, C_2 is the concentration of the diluted sample, and V_2 is the final, total volume of the diluted sample. Rearrange the equation to solve for V_1.

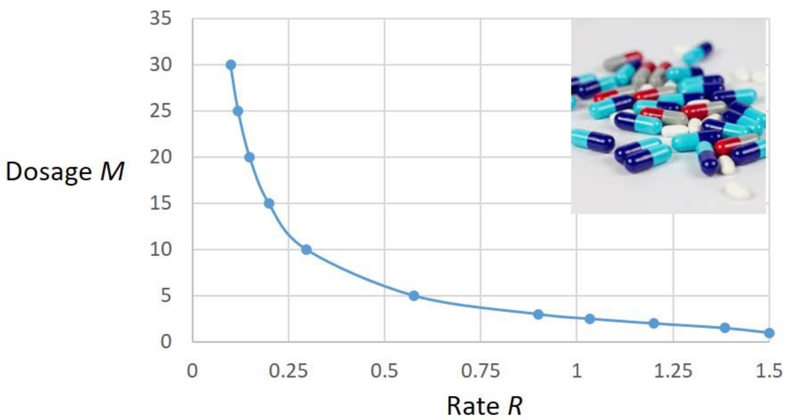
Now use the new formula you obtained to find how many mL of 20% sugar should be used to make 2 mL of 5% sucrose. C_1, then, is equal to 20%, V_1 is the volume you wish to calculate, C_2 is 5%, and V_2 is 2000 µL.

MEDICINE

Suppose an experimental drug changes patient's temperature depending on a dosage M. The following formula relates how fast the temperature drops (rate of change of temperature) to the dosage

$$R_{TEMP} = \frac{3M^2}{M^2+1}.$$

A medical scientist developing the drug needs a formula for computing a dosage M required to achieve a certain rate R so that she can plot a graph of dosage vs rate. Can you assist the scientist with the formula? Can you check with the graph if your formula works for R = 0.125; 0.5; 1?



BUSINESS

You want to go on a celebratory holiday once you have finished school. The trip is estimated to cost €1000. How much do you need to save each month in order to have €1000 in one year's time given that the monthly interest rate is *i*. To answer this question you would need to rearrange the formula

$$A = R\frac{\left[(i+1)^n - 1\right]}{i}$$

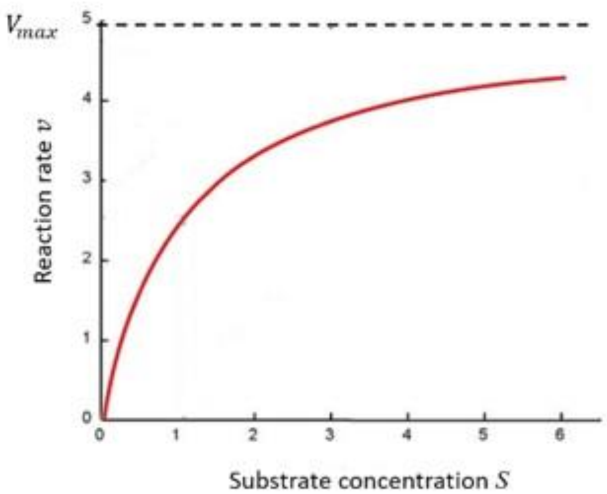
In this formula *A* represents the amount you need to have saved in total, *R* the regular amount you save each month, *i* the interest rate per month and *n* the number of months.

CHEMISTRY & ENVIRONMENTAL SCIENCE

Problem 1. The *Michaelis-Menton equation*, describes the rate of enzymatic reactions in biochemistry. The German biochemist Leonor Michaelis and the Canadian medical researcher Maud Menten, derived the equation while studying the kinetics of an enzymatic reaction where the enzyme, invertase, catalysed the breakdown of sucrose into glucose and fructose.The equation relates the rate of the chemical reaction v, to the concentration of the enzyme (e.g. invertase) and the substrate (e.g. sucrose):

$$v = V_{\max}\frac{S}{K_m + S}$$

where *v* is the rate of the chemical reaction - defined to be the number of moles of product (e.g. glucose and fructose) formed per unit time; V_{max} is the maximum rate of the reaction, *S* is the concentration of the substrate; K_m is referred to as the Michaelis constant and is a measure of the substrate's affinity for the enzyme. At low concentrations of the substrate, *v* is linearly proportional to *S*, but when the concentration of the substrate is high relative to the amount of enzyme, *V* is independent of *S* and will approach the maximum reaction rate, V_{max}.



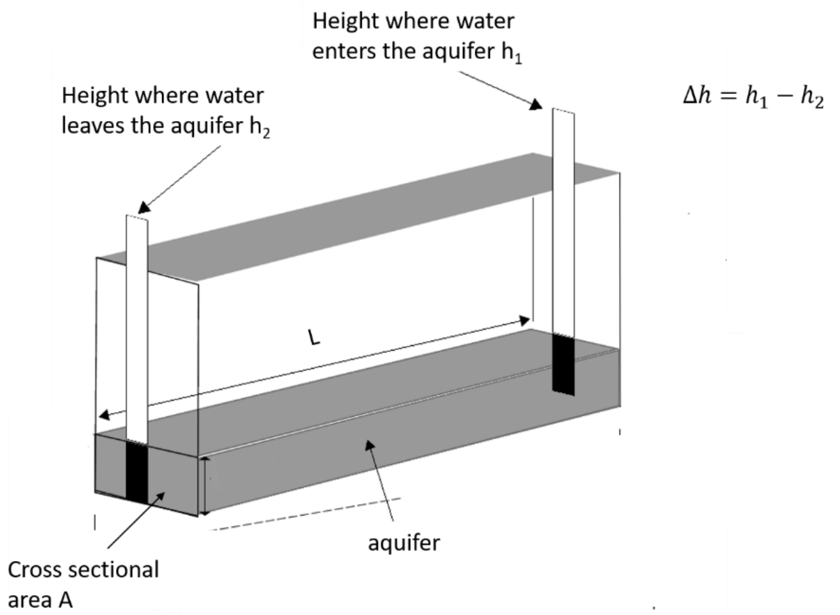
Suppose that you have run an experiment to determine the Michaelis constant for a particular reaction involving an enzyme and substrate. You have measurements for the rate of the chemical reaction *v*, the maximum rate of the reaction *Vmax*, the concentration of the substrate *S.*

Can you rearrange the Micaelis-Menton equation to make K_m the subject of the equation?

Problem 2. An aquifer is a saturated layer of porous geological material that lies under the surface of the Earth. Thirty percent of freshwater on Earth lies below the surface. Aquifers are an important source of drinking water. The rate of flow of water through an aquifer can be described using an equation called Darcy's Law. The equation was derived experimentally by the French engineer Henry Darcy while he was trying to set up a system for filtering water in the city of Dijon.
Water in an aquifer will flow from a high elevation to a low elevation under the force of gravity (water always flows downhill). The rate of flow of water at a particular point is dependent on a number of factors including the gravitational force being applied to the water. The gravitational force is proportional to the difference in heights of the aquifer at the points where the water enters and leaves the aquifer. *Darcy's law* is:

$$Q = K\frac{\Delta h}{L}A$$

where Q represents the rate of flow of water through a cross section of the aquifer (volume per unit time); K represents the hydraulic conductivity of the geological material (distance per unit time); Δh represents the difference in heights (distance); L represents the length of the aquifer (distance); A represents the cross sectional area of the aquifer (area).



The hydraulic conductivity, K, is a constant of proportionality that depends on the geological properties of the material. Materials that water can be transmitted through easily have a large value of K and materials that provide significant resistance to the flow of water have a small value of K. Suppose that an environmental engineer would like to measure the hydraulic conductivity, K, of the material in an underground aquifer. The engineer has measured: the rate of flow of water through the aquifer (Q), the difference in heights where the water enters and leaves the aquifer (Δh), the length of the aquifer and the cross sectional area of the aquifer

Can you rearrange Darcy's law to make K the subject of the equation?

COMPUTER SCIENCE

Problem 1. You are writing some computer code to convert Fahrenheit (*F*) to Celsius (*C*). You Google '*Fahrenheit to Celsius*' and find the following equation:

$$F = \frac{9}{5}C + 32\,.$$

This formula converts Celsius to Fahrenheit. You want to make the inverse conversion, and therefore need to find *C* written in germs of *F*. Rearrange the formula to make *C* the subject, and hence convert 100 F to Celsius.

Problem 2. A software developer is writing some code that takes as input the volume of a sphere *V* and outputs its radius *r*. The developer Googles '*volume of sphere*' and finds the following equation:

$$V = \frac{4}{3}\pi r^3\,.$$

This formula takes *r* as input and gives the volume *V*. The software developer wants to do the opposite, and therefore needs to rearrange the formula so that *r* is written in terms of *V*.

Problem 3. In computer networks, maximum data throughput is a measure of how much data can be sent and received at a time. The maximum data throughput *T* is related to data size *d*, overhead bits *h* (non-data information) and channel bandwidth *b* via the formula:

$$T = \frac{d}{d+h} \times b\,.$$

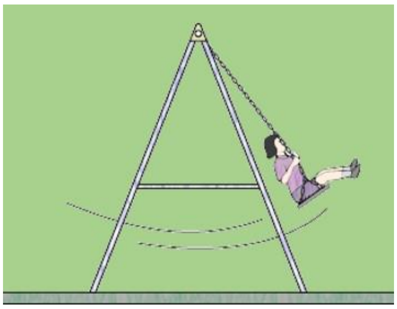
How will this formula change if a network engineer needs to express *h?*

ENGINEERING

Problem 1. A mechanical engineer designing a device working on the principle of a simple pendulum uses the formula:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

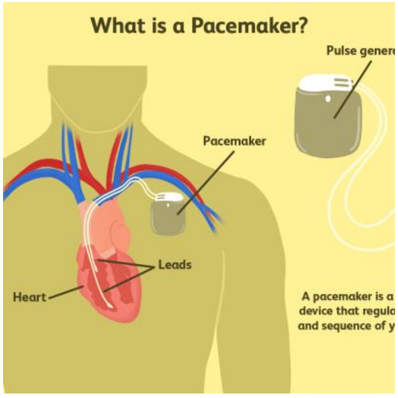
This formula relates the time period *T* to the length of thread *l* and gravitational acceleration *g*. The engineer aims to design a device with a specific time period *T*. Transpose the formula to help the engineer calculate the length *l*.



Problem 2. A pacemaker is a small device that is placed under the patient's skin to help control their heartbeat. An electronic engineer is making an electric circuit that will power and control the pacemaker. The formula

$$P = R \cdot I^2$$

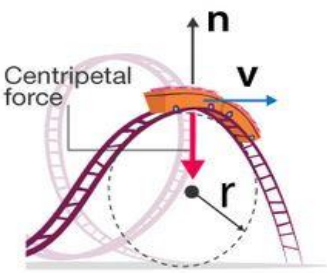
relates the power *P* a circuit generates with the resistance *R* and current *I*. The engineer needs to solve the formula for the current *I* so that they can analyse how the current depends on other parameters of the circuit.



Problem 3. A mechanical engineer is working on a new rollercoaster for amusement parks and is, of course, concerned with its safety. The engineer uses a known formula

$$F = \frac{mv^2}{r}$$

that relates the mass of the cart and its passengers *m*, the radius of the loop *r* and the speed *v* with the force *F* acting on the cart. To generate a table of the maximum mass *m* corresponding to different loop radii the engineer needs make *m* the subject of the formula.



FORENSIC SCIENCE

Problem 1. While crime scene investigators have always been able to determine the direction a blood spatter comes from, they've never been as good with the height—often key for figuring out how a victim was positioned during the attack. A new equation uses simple high school trigonometry and introductory physics to reverse-calculate height by finding an elevation consistent with two blood drops. In the formula below *t*1 and *t*2 are tangents of the angles at which the first and the second blood drops hit the ground, *r*1 and *r*2 are the horizontal distances the first and the second drops travelled, and *Zo* is the height of the blood at the beginning of its parabolic arc, that is, when it left the body.

$$Z_0 = \frac{(\tau_1 - \tau_2)}{2r_2 - 2r_1}$$

Can you rearrange this equation to express the difference between horizontal distances travelled by the first and second drops, i.e. make *r*2- *r*1 the subject of the formula?

Problem 2. A murder has taken place on a sleepy little island off West Cork. You are the newly appointed chief forensic scientist on the case. This is your first case and you are determined to ensure that justice is served. After a ten minute boat ride from the mainland you arrive on the island where the local officer provides you with the following information.. The body temperature of the murder victim was 29ºC when discovered at 10pm and 23ºC two hours later, after the gardaí photographers and investigators had finished their work. The body was found outside where the air temperature was 0ºC. Forensic evidence links a suspect, Mr. X, to the scene of the crime but Mr. X claims he is innocent. There is video tape evidence that shows Mr. X was on the mainland at 8.12pm. Due to the pending trial it is vital for Mr. X's alibi that an estimate of time of death is found.

Could Mr. X have committed the murder on the island? Maths to the rescue! We note that normal body temperature is 37ºC.

Using the information above it can be shown that the temperature of the body *T* (ºC) at time *t* hours after midday is given by the equation

$$T = 92.5e^{-0.116t}.$$

If we fill in the body temperature at the time of death (which is 37ºC) and rearrange the equation to solve for *t*, we can determine the time of death. Can you determine whether Mr. X could have committed the murder or not?